[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : **1096** **C**

Unique Paper Code : 32347508

Name of the Paper : Combinatorial Optimization

Name of the Course : **B.Sc. (H) Computer Science**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All parts of Question 1 (**Part** A) are compulsory.
3. Attempt any **four** questions from **Part B**.
4. All questions in **Part B** carry equal marks.

Section A

1. (a) Differentiate between a Linear Program and an Integer Program with the help of an example. (2)

P.T.O.

(b) How is infeasible solution detected in the simplex method? (3)

(c) Let G be an arbitrary flow network, with source s, sink t, and positive "integer" capacity c(e) for every edge e. Consider a minimum s-t cut (S, T) of G. Now construct G' as the same graph as G and with capacity c'(e) = 2*c(e) (for each edge e). Now consider the cut (S, T) for G'. Is (S, T) a minimum cut for G'? Explain with an example. (3)

(d) Prove that a feasible solution x of a linear program in equational form is basic if and only if columns of matrix A_K are linearly independent where $K = \{j \in \{1,2,\ldots, n\} : x_j > 0\}$ (4)

(e) Convert the following linear program into equational form: (4)

Maximize $4x_1 - 2x_2$

Subject to $x_1 - x_2 \leq 4$

$x_1 + 2x_2 \geq 5$

$x_2 \geq 0$, x_1 unrestricted

(f) Define convex set. Prove that the intersection of arbitrary collection of convex set is convex. (4)

(g) An oil company has two depots A and B with capacities of 7000L and 4000L respectively. The company is to supply oil to three petrol pumps D, 'E and F whose requirements are 4500L, 3000L and 3500L respectively. The distance (in kilometres) between the depots and the petrol pump is given in the following table :

Distance in Km.		
From/To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10L of oil is rupees 1 per kilometre how should the delivery be scheduled in order that transportation cost is minimum? Formulate the above problem as linear program. (5)

(h) Write the dual of following linear programming problem

Maximize $Z = 3X_1 + 2X_2 + 3X_3$

Subject to $2X_1 + X_2 + X_3 = 2$

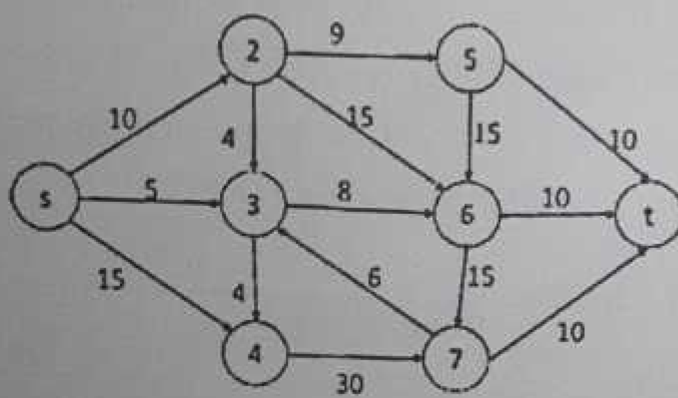
$X_1 + 3X_2 + X_3 \geq 6$

$3X_1 + 4X_2 + 2X_3 \leq 8$

$X_1, X_2, X_3 \geq 0$ (5)

P.T.O.

(i) Delete the best set of edges to disconnect t from s, where best set is defined as the set of edges with total minimum capacity. (5)



Section B

2. (a) State min-cut and max-flow theorem. (3)

(b) A network has edges with distances as shown in the following table

	S	A	B	C	D	E	T
S	-	40	40	-	-	-	-
A	40	-	-	-	15	20	-
B	40	-	-	45	10	-	-
C	-	-	45	-	15	-	50
D	-	15	10	15	-	10	15
E	-	20	-	-	10	-	25
T	-	-	-	50	15	25	-

The letters refer to nodes in the network. Find the shortest path from S to each node using Djikstra's algorithm. (7)

3. Solve the following Linear Programming problem using 2-phase simplex method. (10)

Maximize $Z = 2X_1 + 2X_2 + 4X_3$

Subject to

$$2X_1 + X_2 + X_3 <= 2$$

$$3X_1 + 4X_2 + 2X_3 >= 8$$

$$X_1, X_2, X_3 \geq 0$$

4. (a) Formulate an Integer Program to determine a maximum independent set of the following graph. State LP relaxation condition also.

(5)

(b) An LP has been solved in the standard form and the following optimal solution for the primal and dual problems have been determined. The complementary pairs have been listed next to each other.

Primal	x_1 : 10	x_2 : ?	x_3 : ?	x_4 : 5	x_5 : 4	x_6 : ?
Dual	y_4 : ?	y_5 : 5	y_6 : ?	y_1 : 0	y_2 : 0	y_3 : 2

P.T.O.

The Primal objective is given by x_3 and the Dual objective is given by $y_1 + y_2 + 2y_3$. Which of the following statements regarding the missing parts in the table are correct?

(i) $y_4 = 0$

(ii) $x_2 = 0$

(iii) x_3 may or may not be zero. We do not have sufficient data to conclude.

(iv) $x_6 = 0$

(v) $x_3 = 4$ (5)

5. (a) Can the dual of an unbounded primal LP be unbounded? Explain. (3)

(b) Consider the following LPP:

Min $Z = 3X_1 + 8X_2$

S.T.

$X_1 + X_2 \geq 8$

$2X_1 - 3X_2 \leq 0$

$X_1 + 2X_2 \leq 30$

$3X_1 - X_2 \geq 0$

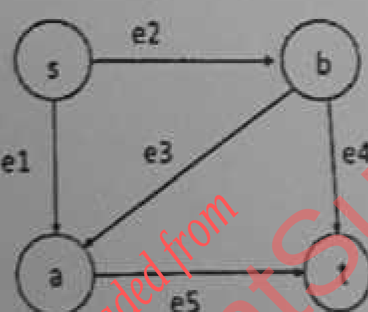
$X_1 \leq 10$

$X_2 \geq 9$

$X_1, X_2 \geq 0$

Solve the above LPP graphically and find the optimal solution using the method of iso-profit line method. (7)

6. (a) Consider the following flow graph. Let the capacity and current flow of edge e be denoted by c(e) and f(e) respectively. Further, let $c(e_1)=5$, $f(e_1)=2$, $c(e_2)=3$, $f(e_2)=2$, $c(e_3)=2$, $f(e_3)=0$, $c(e_4)=4$, $f(e_4)=2$, $c(e_5)=2$, $f(e_5)=2$. Determine the max flow and min cut. (5)



(b) For the following LP, prove or disprove that $x = [1,1,1]^T$ is an optimal solution by using the complementary slackness conditions.

Min $-5x_1 - 4x_2 - 3x_3$

Subject to $2x_1 + 3x_2 + 1x_3 \geq 5$

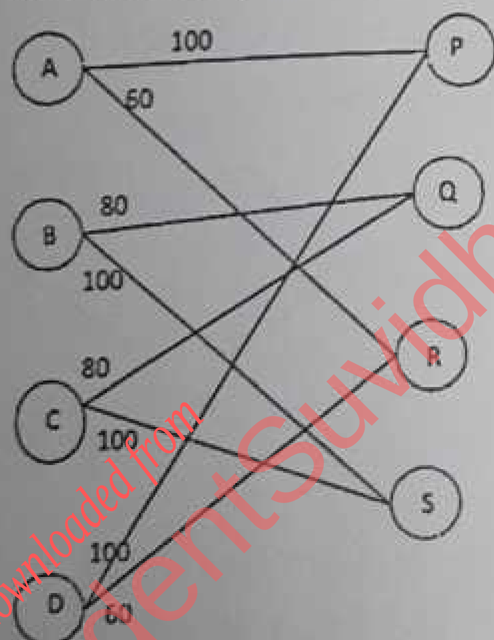
$4x_1 + 1x_2 + 2x_3 \leq 13$

$3x_1 + 4x_2 + 2x_3 = 9$

$x_1, x_2, x_3 \geq 0$ (5)

P.T.O.

7. (a) Consider the fractional solution for maximum weight matching given below : $X_{AP}=0.9$, $X_{AR}=0.1$, $X_{BQ}=0.3$, $X_{BS}=0.7$, $X_{CQ}=0.7$, $X_{CS}=0.3$, $X_{DP}=0.1$, $X_{DR}=0.9$. Apply cycle cancelling procedure to above solution to obtain an integral solution. Show all steps. Compare it with fractional solution. Can we obtain a non-integral optimal solution by solving the above LP? Justify your answer. (5)



(b) Let P be the set of all feasible solutions of a linear program in equational form, Then, show that the following two conditions for a point v ∈ P are equivalent :

(a) v is a vertex of the polyhedron P.

(b) v is a basic feasible solution of the linear program. (5)

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